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An effective description of the landscape — I

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ABSTRACT: We study under what conditions massive fields can be "frozen" rather than integrated out in certain four dimensional theories with global or local $\mathcal{N} = 1$ supersymmetry. We focus on models without gauge fields, admitting a superpotential of the form $W = W_0(H) + \epsilon W_1(H, L)$, with $\epsilon \ll 1$, where H and L schematically denote the heavy and light chiral superfields. We find that the fields H can always be frozen to constant values H_0, if they approximately correspond to supersymmetric solutions along the H directions, independently of the form of the Kähler potential K for H and L, provided K is sufficiently regular. In supergravity W_0 is required to be of order ϵ at the vacuum to ensure a mass hierarchy between H and L. The backreaction induced by the breaking of supersymmetry on the heavy fields is always negligible, leading to suppressed F^H-terms. For factorizable Kähler potentials W_0 can instead be generic. Our results imply that the common way complex structure and dilaton moduli are stabilized, as in Phys. Rev. **D 68** (2003) 046005 by Kachru et al., for instance, is reliable to a very good accuracy, provided W_0 is small enough.

KEYWORDS: Supergravity Models, Supersymmetric Effective Theories, Supersymmetry Breaking, Superstring Vacua.

Contents

1. Introduction

Effective field theories are probably the main tool of investigation in particle physics. Thanks to the decoupling of massive particles in low energy processes at the quantum level [1], effective theories provide a significant and reliable simplification in the description of most physical systems. In a standard quantum field theory perturbative context, where the location of the vacuum and the structure of the tree level Lagrangian is typically simple to determine, one commonly integrates out heavy fields to get an effective field theory where processes at the classical and quantum level are efficiently computed.

The situation is a bit different in string theory, where the number of degrees of freedom is so large (actually infinite), that massive particles are typically neglected, rather than integrated out. At a first approximation this is fine, provided such states are easily detected and sufficiently heavy, which is the case for string excitations or Kaluza-Klein states in a given compactification. Below the string or compactification scale, the remaining "light" states are typically described by a given supergravity (SUGRA) field theory, assumed from now on to be four-dimensional with $\mathcal{N} = 1$ supersymmetry (SUSY). Despite this huge simplification, the resulting SUGRA field theory typically contains hundreds of fields, making an explicit full study of these theories a formidable task. Thanks to the recent progress in string compactifications with fluxes, many of these fields, which are moduli fields in an adiabatic approximation where fluxes are turned off, can get large masses and

one can hope to get rid of them, in the spirit of effective theories. The scalar potential associated to these theories is however very complicated, admitting in general a "landscape" of possible vacua [2], and even just determining the location of the vacua is often very hard. As a matter of fact, in most cases a large subset of the moduli are neglected without a proper justification. They are assumed to get a sufficiently large mass by a tree-level flux-induced superpotential or somehow have suppressed couplings with the remaining light fields, whereas the general correct procedure would consist in integrating them out from the action and then check whether the generated effective couplings are negligible or not.

In simple global SUSY theories a heavy chiral superfield H is easily integrated out at tree-level and at low energies (when derivatives interactions are negligible) in unbroken SUSY vacua, by just taking $\partial_H W = 0$ and plugging back the solution in the superpotential W and the Kähler potential K. In this way one gets an effective holomorphic superpotential and Kähler potential for the remaining fields, see e.g. [3].[1] This procedure is easily implemented in the SUSY theories with a simple vacuum structure, but not in the complicated SUGRA theories we are interested in, since it assumes the knowledge of the vacuum and of the mass spectrum, which are actually the first quantities to be determined. Even if we have some guess about which are the heavy fields H in a given region in moduli space of the theory, their Vacuum Expectation Values (VEV's) will in general depend on the remaining light fields L, so that already at the vacuum level we encounter problems to decouple heavy and light degrees of freedom. In this situation, it is not well defined the notion of "freezing" the heavy moduli H, namely fixing them at their VEV's and neglecting their quantum fluctuations, and practically impossible to properly integrate them out. As a further complication, integrating out H in SUGRA generally would give rise to a non-holomorphic and quite complicated effective superpotential for the light fields L, as emphasized in [5]. The non-holomorphicity of the superpotential is not in contradiction with SUSY, since the splitting between K and W in SUGRA is arbitrary, being $G = K + \log |W|^2$, the only Kähler invariant quantity, but certainly the resulting effective theory will generally be quite awkward.

Aim of the present paper is to study under what conditions massive fields can be frozen rather than properly integrated out, and yet get a reliable effective theory, in 4D theories with $\mathcal{N} = 1$ global or local supersymmetries. We focus on this paper on theories with only chiral multiplets and no gauge fields. In a companion paper, we will extend the present results to the more interesting, but more involved, case where gauge fields are added [6]. As mentioned before, a meaningful freezing of the heavy fields requires that in some approximation their VEV's do not depend on the light fields and can be fixed to constant values H_0, independently of the light field dynamics. We are then led to study theories with a superpotential of the form

$$W(H, L) = W_0(H) + \epsilon \, W_1(H, L) \,, \tag{1.1}$$

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[1]Ref. [4] found that the equation $\partial_H W = 0$ in general receives corrections. We believe that the corrections found in [4] are automatically taken into account in substituting the solution for $\partial_H W = 0$ back in the Kähler potential as well, in which case $\partial_H W = 0$ is correct and always leads to the correct effective potential.

with $\epsilon \ll 1$, where H and L schematically denote heavy and light chiral superfields.[2] The Kähler potential is arbitrary, with the only assumption that all the eigenvalues of the associated Kähler metric are parametrically larger than ϵ. By expanding the theory in a series in ϵ, we find that for theories with global SUSY the fields H can always be reliably frozen to constant values H_0 (defined as the approximate SUSY solutions $F_{H,0} = \partial_H W_0 = 0$) independently of the form of the Kähler potential K for H and L. By reliably, we mean that the *complete* bosonic effective low-energy theory obtained by freezing the fields H is identical, at leading order in ϵ, to the full bosonic effective theory obtained by classically integrating H out at the gaussian level at low energies, i.e. neglecting their kinetic terms. In the following, for simplicity of language, we will denote the former and latter "simple" and "full" effective theories, respectively. This result applies on any vacuum, supersymmetric or not. In SUGRA theories the covariant derivative entering in the definition of the F term would lead to the condition $F_{H,0} = \partial_H W_0 + (\partial_H K) W_0 = 0$, rather than $\partial_H W_0 = 0$. The former equation depends in general on the light fields through K and does not allow for a well-defined "freezing" for the fields H. An extra requirement is hence needed. Either i) the Kähler potential approximately factorizes, $K = K_H(H) + K_L(L) + O(\epsilon)$, so that $F_{H,0} = 0$ does not depend on L at leading order in ϵ or ii) $W_0(H_0) \lesssim O(\epsilon)$, so that effectively $F_{H,0} = \partial_H W_0 = 0$. A factorisable Kähler potential, together with a superpotential of the form (1.1), fulfills actually a sufficient condition for decoupling found in [7] (see also [8]). For this reason, we mainly focus on the second possibility, where the Kähler potential is generic. The smallness of $W_0(H_0)$ is well justified from the point of view of an effective theory of light fields, given the rather universal SUGRA contributions to the masses, proportional to $\langle W \rangle$. From a phenomenological point of view, this is related to the requirement of low energy spontaneous supersymmetry breaking and a not too large gravitino mass, given by

$$m_{3/2} = e^{K/2} |W| . \tag{1.2}$$

We are eventually interested in vacua where SUSY is spontaneously broken, so another important effect that has to be considered is the backreaction of the SUSY breaking sector on the heavy fields. Eq. (1.1) and $F_{H,0} = 0$ generally predicts

$$F_L \sim F_H \sim O(\epsilon) . \tag{1.3}$$

Interestingly enough, independently of the form of K, the upper F term component are

$$F^L \sim O(\epsilon) , \quad F^H \sim O(\epsilon^2) , \tag{1.4}$$

so that at leading order the backreaction of SUSY breaking on the heavy sector is negligible, in agreement with the result that the fields H can reliably be frozen at an approximately SUSY solution along the H directions.[3] These results are valid for any value of the cosmological constant.

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[2]This distinction is independent of the Kähler potential. As we will see, it is always possible to define a canonically normalized field basis where the fields H give rise to the canonical heavy field fluctuations.

[3]This can also be seen by noting that the SUSY transformations of the heavy fermions are proportional to F^H.

A natural class of string models where our general results can be applied are IIB/F-theory Calabi-Yau compactifications with fluxes, where W_0 is identified with the Gukov-Vafa-Witten flux superpotential [9], generally able to fix all complex structure moduli, including the dilaton [10]. Fixing complex structure moduli is actually the first step of the Kachru-Kallosh-Linde-Trivedi (KKLT) procedure [11] in finding de Sitter (dS) SUGRA vacua with stabilized moduli in string theory. The complex structure moduli and the dilaton can collectively be identified with our fields H. The Kähler structure moduli, as well as other fields possibly responsible for SUSY breaking, can be identified with the fields L. In this case, $W_1(L, H)$ includes non-perturbatively generated superpotential terms for the Kähler moduli. Our results prove that H can naively and reliably be frozen by just using W_0, as [11] does. One can then consistently forget about the complex structure and dilaton dependence appearing in W_1, possible Kähler mixing between H and L, which do typically appear at some order in α' or g_s, and about how the fields H contribute to the SUSY breaking mechanism induced by the fields L, provided $W_0 \sim \epsilon$ at the vacuum.[4] In [11] the tuning $W_0 \ll 1$ was actually required for another reason, namely to supersymmetrically stabilize the universal Kähler modulus to large values. We find here that the smallness of W_0 is more important than previously thought, being the basis for a consistent decoupling of the two moduli sectors. One has also to check that the mass matrix of the H fields, governed by W_0, is positive definite with all eigenvalues parametrically larger than the effective ϵ defined by W_1.[5] As a matter of fact, previous works pointing out problems in the way KKLT stabilize complex structure and dilaton moduli boil down to not satisfy either the requirement $W_0 \ll 1$ [5], or the requirement that all the heavy moduli get large positive SUSY mass terms [13]. Interestingly enough, one typically requires many complex structure moduli to be able to tune W_0 and the cosmological constant [14] to sufficiently small values, in which case there is no reason to expect small or negative SUSY mass terms [15, 16].[6] It is important to remark that our results are valid in the context of a spontaneous SUSY breaking mechanism. In this sense, they cannot automatically provide a solid framework for the whole KKLT procedure, since it is not yet clear whether the SUSY breaking mechanism advocated in [11] (the addition of $\bar{D}_3$-branes) admits an interpretation in terms of a spontaneously broken 4D $\mathcal{N} = 1$ SUGRA theory.

Although we mostly focus on SUGRA models with $W_0 \sim \epsilon$, we also briefly study how eq. (1.1) leads to decoupling, when W_0 is generic, but K is almost factorizable. We then consider in some detail a relevant model of this sort, namely a so called large volume model [17, 18] arising from flux compactifications of Type IIB string theory on Calabi-Yau (CY) 3-folds with an exponentially large volume. The superpotential associated to this model is effectively of the form (1.1), with ϵ related to the inverse volume of the CY. There is not a mass hierarchy between the heavy and light fields, but nevertheless the couplings between the two sectors are suppressed, so that an effective decoupling occurs, namely the

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[4]If the Kähler potential happens to be almost factorizable, the condition $W_0 \sim \epsilon$ might be partially relaxed, since in this case a sort of mixing between the decoupling effects i) and ii) discussed before apply.

[5]See also [12] where the condition $W_0 \ll 1$ has been shown to be necessary to have a reliable description of the vacuum in the effective theory.

[6]We thank S. Kachru for bringing [15] to our attention.

full and simple effective potentials agree at leading order in ϵ.

The structure of the paper is as follows. In section 2 we introduce the basic elementary tools used throughout the paper in a simple non-supersymmetric bosonic σ-model. In section 3 we derive the equivalence between the full and the simple effective actions for a generic SUSY model in flat space and with a superpotential of the form (1.1). In section 4 we show how the analysis straightforwardly applies to SUGRA, provided W_0 is small at the minimum for a generic Kähler potential or the latter is almost factorizable and W_0 generic. In section 5 we conclude. We explicitly work out in three appendices some concrete models for illustrative purposes. In appendix A we study a very simple model with global SUSY, in appendix B a more complicated KKLT-like model and finally, in appendix C, we report the study of a large volume model as a specific example of a SUGRA model with an almost factorizable Kähler potential.

2. Non-supersymmetric σ-model

Before considering the more complicated SUSY case, we establish a very simple, yet useful, result valid for an arbitrary non-supersymmetric bosonic σ-model. Let us consider a system of $n_H + n_L$ interacting real scalar fields H^i and L^α, $i = 1, \ldots, n_H$, $\alpha = 1, \ldots, n_L$, with Lagrangian density

$$\mathcal{L} = \frac{1}{2} g_{MN}(\phi^M) \partial \phi^M \partial \phi^N - V(\phi^M), \tag{2.1}$$

and potential

$$V(\phi^M) = V_0(H^i) + \epsilon V_1(H^i, L^\alpha), \tag{2.2}$$

where $M = 1, \ldots, n_H, 1 + n_H, \ldots, n_H + n_L$, $\phi^M = (H^i, L^\alpha)$ and $\epsilon \ll 1$. Notice that the splitting between the fields H^i and L^α is dictated by V_0, namely we call H^i (the "heavy fields") the ones appearing in V_0. We assume that at a given vacuum $\langle \phi^M \rangle$, the metric g_{MN} is non-singular and $\partial_i \partial_j V_0$ is positive definite with all eigenvalues parametrically larger than ϵ. Under these assumptions, at leading order in ϵ, the simple effective low-energy Lagrangian associated to $\mathcal{L}$ is

$$\mathcal{L}_{\text{sim}} = \frac{1}{2} g_{\alpha\beta}(L^\alpha, H_0^i) \partial L^\alpha \partial L^\beta - \left[V_0(H_0^i) + \epsilon V_1(H_0^i, L^\alpha) \right], \tag{2.3}$$

where H_0^i are the leading order VEV's for H^i, independent of L^α, satisfying $\partial_i V_0(H_0^j) = 0$. We want to show that $\mathcal{L}_{\text{sim}}$ provides the correct effective Lagrangian for arbitrary kinetic mixing terms at leading order in an expansion in ϵ. First of all, we look for (space-time independent) vacua of $\mathcal{L}$ by studying the extrema of the potential V in the original, non-canonically normalized, field basis given by H^i and L^α. Let H_0^i be the solutions to $\partial_i V_0 = 0$ at $O(\epsilon^0)$. At this order, L^α are undetermined, since $\partial_\alpha V_0$ trivially vanishes. At $O(\epsilon)$, we get the leading VEV's L_0^α for the light fields from $\partial_\alpha V_1(H_0^i) = 0$ and the first corrections H_1^i to the VEV's of the heavy fields from the linear equations $\partial_i \partial_j V_0(H_0^k) H_1^j + \partial_i V_1(H_0^k, L_0^\alpha) = 0$. Once the approximate vacuum (H_0^i, L_0^α) has been determined, we can diagonalize the metric in order to identify the canonically normalized field fluctuations $\hat{\phi}_c$ starting from

the field fluctuations $\hat{\phi} = \phi - \langle \phi \rangle$.[7] At leading order the matrix to diagonalize is $g_{MN,0} = g_{MN}(H_0^i, L_0^\alpha)$ and, for simplicity, we assume all its eigenvalues to be parametrically larger than ϵ. A positive definite real symmetric matrix can always be written as the product of a lower triangular matrix times its transpose.[8] We then write $g_0 = (T^{-1})^t T^{-1}$, so that $\hat{\phi} = T\hat{\phi}_c$, where

$$T = \begin{pmatrix} (T_H)_j^i & 0 \\ (T_{HL})_i^\alpha & (T_L)_\beta^\alpha \end{pmatrix} \tag{2.4}$$

and $\hat{\phi} = (\hat{H}^i, \hat{L}^\alpha)^t$. In this new basis, the Lagrangian (2.1) reads

$$\mathcal{L} = \left[\frac{1}{2} + O(\epsilon) \right] \left[(\partial \hat{H}_c^i)^2 + (\partial \hat{L}_c^\alpha)^2 \right] + \ldots +$$
$$\left[V_0(H_0^i + \epsilon H_1^i + (T_H \hat{H}_c)^i) + \epsilon V_1(H_0^i + \epsilon H_1^i + (T_H \hat{H}_c)^i, L_0^\alpha + (T\hat{\phi}_c)^\alpha) + O(\epsilon^2) \right], \tag{2.5}$$

where the ellipsis in eq. (2.5) stands for all the higher order terms arising from the expansion of the metric in quantum fluctuations, the specific form of which are not needed. Thanks to the triangular form of T, $\hat{H}_c^i$ are linear combinations of the $\hat{H}^i$ only. The next step would be to diagonalize the mass matrix of the heavy fields, but it will not be explicitly needed. Indeed, we see from the term V_0 in eq. (2.5) that the fields $\hat{H}_c^i$ have all a leading mass term of $O(\epsilon^0)$, the $n_H \times n_H$ mass matrix being of the form $M^2 = T_H^t M_0^2 T_H$, where $(M_0^2)_{ij} = \partial_i \partial_j V_0(H_0^i)$. Since by assumption M_0^2 is positive definite, so it is M^2. Due to the form of the potential in eq. (2.5), integrating out the fluctuations $\hat{H}_c^i$ at quadratic level will only affect the effective theory at $O(\epsilon^2)$, so we can simply set $\hat{H}_c^i = 0$ if we want a reliable Lagrangian up to $O(\epsilon)$. We can go back to non-canonically normalized fields $\hat{L}^\alpha = (T_L^{-1} \hat{L}_c)^\alpha$. Since $T_L^{-1} = (T^{-1})_\beta^\alpha$, the kinetic mixing matrix reads now $(T_L^{-1})^t (T_L)^{-1} = g_{\alpha\beta,0}$ and all the terms in the ellipsis in eq. (2.5), when $\hat{H}_c^i = \hat{H}^i = 0$, simply reproduce the full field-dependent metric $g_{\alpha\beta}(L^\alpha, H_0^i)$ appearing in eq. (2.3). The resulting full effective Lagrangian becomes

$$\mathcal{L}_{\text{full}} = \left[\frac{1}{2} g_{\alpha\beta}(L^\alpha, H_0^i) + O(\epsilon) \right] \partial L^\alpha \partial L^\beta - \left[V_0(H_0^i + \epsilon H_1^i) + \epsilon V_1(H_0^i, L^\alpha) + O(\epsilon^2) \right]$$

and hence, modulo irrelevant constant terms, at leading order in ϵ we get the desired result

$$\mathcal{L}_{\text{sim}} = \mathcal{L}_{\text{full}}. \tag{2.6}$$

3. Supersymmetric σ-model

The results found in the previous section, being of general validity, apply to SUSY theories as well. But the structure of the scalar potential fixed by SUSY allows for a more powerful result, valid up to $O(\epsilon^2)$. Neglecting gauge fields, a SUSY theory is specified by a Kähler potential $K(\phi, \bar{\phi})$, taken to be generic, and a superpotential, taken as in eq. (1.1):

$$W(H^i, L^\alpha) = W_0(H^i) + \epsilon W_1(H^i, L^\alpha), \tag{3.1}$$

---

[7]We thank A. Romanino for essentially providing us the argument that follows.

[8]This procedure, called Cholesky decomposition, is unique if we require the diagonal entries of T to be strictly positive. See e.g. [19].

using the same conventions as before, but considering that now H^i and L^α are complex (super)fields.[9] The scalar potential of the theory is

$$V = g^{\bar{M}N}\overline{F}_{\bar{M}}F_N \,, \tag{3.2}$$

with $F_M = \partial_M W$ and $g^{\bar{M}N}$ the inverse matrix of $g_{M\bar{N}} = \partial_M \partial_{\bar{N}} K$. The potential V is a sum of three terms when expanded in ϵ: $V = V_0 + \epsilon V_1 + \epsilon^2 V_2$, with

$$
\begin{aligned}
V_0 &= g^{\bar{j}i}\overline{F}_{\bar{j},0}F_{i,0} \,, \\
V_1 &= g^{\bar{j}i}\overline{F}_{\bar{j},1}F_{i,0} + g^{\bar{\alpha}i}\overline{F}_{\bar{\alpha},1}F_{i,0} + c.c. \,, \\
V_2 &= g^{\bar{j}i}\overline{F}_{\bar{j},1}F_{i,1} + g^{\bar{\beta}\alpha}\overline{F}_{\bar{\beta},1}F_{\alpha,1} + (g^{\bar{\alpha}i}\overline{F}_{\bar{\alpha},1}F_{i,1} + c.c.) \,,
\end{aligned}
\tag{3.3}
$$

where $F_M = F_{M,0} + \epsilon F_{M,1}$ and

$$
\begin{aligned}
F_{i,0} &= \partial_i W_0 \,, & F_{\alpha,0} &= 0 \,, \\
F_{i,1} &= \partial_i W_1 \,, & F_{\alpha,1} &= \partial_\alpha W_1 \,.
\end{aligned}
\tag{3.4}
$$

We assume that at the vacuum the metric $g_{M\bar{N}}$ is positive definite and that $\partial_i \partial_j W_0$ is non-degenerate with eigenvalues parametrically larger than ϵ. We now establish that if the heavy fields sit at a SUSY vacuum at leading order in ϵ, the bosonic low energy effective theory of the light fields L^α is described by the simple SUSY effective theory with $K_{\rm sim}(L^\alpha, \bar{L}^\alpha) = K(H_0^i, L^\alpha, \bar{H}_0^i, \bar{L}^\alpha)$, $W_{\rm sim} = W_0(H_0^i) + \epsilon W_1(H_0^i, L^\alpha)$:

$$\mathcal{L}_{\rm sim} = \tilde{g}_{\alpha\bar{\alpha}}\partial L^\alpha \partial \bar{L}^{\bar{\alpha}} - V_{\rm sim}(L^\alpha, \bar{L}^{\bar{\alpha}}) \,, \tag{3.5}$$

with $V_{\rm sim} = \tilde{g}^{\bar{\alpha}\alpha}\tilde{F}_\alpha \overline{\tilde{F}}_{\bar{\alpha}}$, $\tilde{g}_{\alpha\bar{\alpha}} = \partial_\alpha \partial_{\bar{\alpha}} K_{\rm sim}$ and $\tilde{F}_\alpha = \partial_\alpha W_{\rm sim}$.

Let us start by finding the vacuum in an expansion in ϵ:

$$\langle \phi^M \rangle = \phi_0^M + \epsilon \phi_1^M + \epsilon^2 \phi_2^M + \dots \,. \tag{3.6}$$

The equations of motion (e.o.m.) up to $O(\epsilon^2)$ read

$$(\partial_M V)_0 = \partial_M V_0 = 0 \,, \tag{3.7}$$

$$(\partial_M V)_1 = \partial_M V_1 + (\partial_M \partial_N V_0)\phi_1^N + (\partial_M \partial_{\bar{N}} V_0)\bar{\phi}_1^{\bar{N}} = 0 \,, \tag{3.8}$$

$$
\begin{aligned}
(\partial_M V)_2 = {}&\partial_M V_2 + (\partial_M \partial_N V_1)\phi_1^N + (\partial_M \partial_{\bar{N}} V_1)\bar{\phi}_1^{\bar{N}} + (\partial_M \partial_N V_0)\phi_2^N + (\partial_M \partial_{\bar{N}} V_0)\bar{\phi}_2^{\bar{N}} \\
&+ \frac{1}{2}(\partial_M \partial_N \partial_P V_0)\phi_1^N \phi_1^P + \frac{1}{2}(\partial_M \partial_{\bar{N}} \partial_{\bar{P}} V_0)\bar{\phi}_1^{\bar{N}} \bar{\phi}_1^{\bar{P}} + (\partial_M \partial_N \partial_{\bar{P}} V_0)\phi_1^N \phi_1^{\bar{P}} = 0,
\end{aligned}
\tag{3.9}
$$

where all quantities in eqs. (3.7), (3.8) and (3.9) are evaluated at $\phi^M = \phi_0^M$. At $O(\epsilon^0)$, the equations $(\partial_k V)_0 = 0$ can generally admit both SUSY and non-SUSY solutions. Contrary to the latter ones, which can depend on L^α through the inverse metric components $g^{\bar{j}i}$, the SUSY solutions depend on H^i only. Let H_0^i be the SUSY solutions:

$$F_{i,0}(H_0^i) = \overline{F}_{\bar{i},0}(\bar{H}_0^i) = 0 \,. \tag{3.10}$$

---

[9]For simplicity of notation, below and throughout the paper, we use the same notation to denote a chiral superfield and its lowest scalar component, since it should be clear from the context to what we are referring to. Similarly, we will mostly not report the complex conjugate VEV's of the fields, their corresponding equations of motion, and so on.

The e.o.m. $(\partial_\alpha V)_0 = 0$ are identically satisfied when $H^i = H_0^i$, so that L^α are not determined at this order. Interestingly enough, the e.o.m. of the light fields at $O(\epsilon)$, $(\partial_\alpha V)_1 = 0$, are automatically satisfied as well, since

$$\partial_\alpha \partial_M V_0 = \partial_\alpha \partial_{\bar{M}} V_0 = \partial_\alpha V_1 = 0\,, \tag{3.11}$$

when evaluated at $H = H_0^i$, as can easily be checked using the explicit expressions in eq. (3.3). The displacement of the heavy field VEV's at $O(\epsilon)$ is calculated by taking $M = j$ in eq. (3.8). We get

$$H_1^i = -(K^{-1})_{\bar{j}}^i \overline{F}_1^{\bar{j}}\,, \tag{3.12}$$

with $K_j^{\bar{i}} = g^{\bar{i}k}\partial_k\partial_j W_0$ and $F_1^i = \overline{F}_{\bar{M},1}g^{\bar{M}i}$, evaluated again at $H^i = H_0^i$. Notice that H_1^i are in general a function of both L^α *and* $\bar{L}^{\bar{\alpha}}$, which are yet to be determined at this order. Hence, up to $O(\epsilon)$, eq. (3.12) is not only valid at the vacuum but is a field identity[10]

$$H_1^i(x) = -(K^{-1})_{\bar{j}}^i \overline{F}_1^{\bar{j}}(H_0^i, L^\alpha(x))\,. \tag{3.13}$$

This is a crucial property to quickly establish the equivalence of the full and simple effective actions, as we will show. At $O(\epsilon^2)$ we finally get non-trivial e.o.m. for L^α as well as the 2nd order displacement of the heavy fields H_2^i, whose explicit form will not be needed. Both H_2^i and L_0^α arise at $O(\epsilon^2)$, but the e.o.m. of L^α do not depend on H_2^i, as can be seen from eqs. (3.9) and (3.11). By plugging eq. (3.12) in eq. (3.9) and after some algebra, we could establish that the e.o.m. that determine L_0^α in the full theory are the same as the one obtained in the simple theory where H^i are frozen to their leading values H_0^i. The best and more instructive way to proceed, however, is by finding the leading power in ϵ of the F terms and their derivatives, evaluated at the shifted vacuum $H_0^i + \epsilon H_1^i$. We get

$$
\begin{array}{cccc}
F_i = O(\epsilon), & F_\alpha = O(\epsilon), & F^i = O(\epsilon^2), & F^\alpha = O(\epsilon)\,, \\
\partial_j F_i = O(1), & \partial_i F_\alpha = O(\epsilon), & \partial_\beta F_i = O(\epsilon), & \partial_\beta F_\alpha = O(\epsilon)\,.
\end{array} \tag{3.14}
$$

Interestingly enough, although in general the backreaction of the light fields on the heavy ones induce F_i-terms of $O(\epsilon)$, the upper components F^i are vanishing at this order since

$$\overline{F}^{\bar{i}}(H_0^i + \epsilon H_1^i) = \left[g^{\bar{i}M}F_{M,1}(H_0^i) + g^{\bar{i}j}\partial_k F_{j,0}H_1^k \right]\epsilon + O(\epsilon^2) \tag{3.15}$$

and the first two terms in eq. (3.15) exactly cancel, due to eq. (3.12). This implies that at linear order the F_i and F_α terms are related as follows:

$$F_i = -\tilde{g}_{i\bar{j}}g^{\bar{j}\alpha}F_\alpha + O(\epsilon^2)\,, \tag{3.16}$$

with $\tilde{g}_{i\bar{j}}$ the inverse metric of $g^{\bar{j}i}$, not to be confused with $g_{i\bar{j}}$. Using eq. (3.15) and the relation (3.16), after some straightforward algebra one finds the desired identification

$$\partial_\alpha V = \partial_\alpha V_{\text{sim}} + O(\epsilon^3)\,, \tag{3.17}$$

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[10]Since we are neglecting gradient term contributions arising when H_1 is space-time dependent, eq. (3.13) is only valid at low energies, when L^α is slowly varying, which is the case of interest.

which implies that the location of the vacuum is reliably computed in the simple theory. In order to establish eq. (3.17), it is very useful to use the matrix identity

$$\tilde{g}^{\bar{\alpha}\alpha} = g^{\bar{\alpha}\alpha} - g^{\bar{\alpha}i}\tilde{g}_{i\bar{j}}g^{\bar{j}\alpha}, \tag{3.18}$$

where $\tilde{g}^{\bar{\alpha}\alpha}$ is the inverse of $g_{\alpha\bar{\alpha}}$, appearing in V_{sim}, not to be confused with $g^{\bar{\alpha}\alpha}$. Thanks to eq. (3.18), in particular, it is easy to show that $F^\alpha = \tilde{F}^\alpha$ at leading order in ϵ.

The full leading order equivalence of the naive low-energy effective theory with the full one proceeds along the same lines of the general non-SUSY case discussed before. Once found the leading order VEV's ϕ_0^M, we write $g_0 = (T^{-1})^\dagger T^{-1}$ (which is the generalization of the Cholesky decomposition for hermitian matrices) and get the canonically normalized field fluctuations $\hat{\phi}_c^M$ as $\hat{\phi} = T\hat{\phi}_c$, $\hat{\bar{\phi}} = T^*\hat{\bar{\phi}}_c$. At quadratic order in the fluctuations, the leading potential term V_0 in eq. (3.3) depends on $\hat{H}_c^i$ only. The latter are hence identified as the proper heavy fields fluctuations, with a mass matrix $T_H^\dagger M^2 T_H$ and

$$M_{ij}^2 = \partial_i \partial_{\bar{k}} \overline{W}_0 g_0^{\bar{k}l} \partial_l \partial_j W_0. \tag{3.19}$$

By assumption, the mass matrix (3.19) has eigenvalues parametrically larger than ϵ. We can expand the scalar potential up to quadratic order in $\hat{H}_c^i$ and hence integrate the heavy fluctuations out. Schematically the expansion is as follows

$$V(H, L) = V(\langle H \rangle, L) + \partial_H V(\langle H \rangle, L)T_H \hat{H}_c + \partial_H^2 V(\langle H \rangle, L)(T_H \hat{H}_c)^2 + O(\hat{H}_c^3). \tag{3.20}$$

Since $\partial_H V$ is at most of $O(\epsilon)$ and $\partial_H^2 V$ of $O(1)$, $\hat{H}_c$ is $O(\epsilon)$. In order to get the effective Lagrangian up to $O(\epsilon^2)$, it is enough to determine the $O(1)$ terms of $\partial_H^2 V$ and the $O(\epsilon)$ terms of $\partial_H V$. Using eqs. (3.14) and (3.16), it is straightforward to show that the linear term in $\hat{H}_c^i$ vanishes, being proportional to $F^i = O(\epsilon^2)$. Hence, integrating out $\hat{H}_c^i$ just amounts to set them to zero! The full effective scalar potential for the light fields $\hat{L}_c^\alpha$ is easily determined. Both $V_0(\hat{L}_c^\alpha)$ and $V_1(\hat{L}_c^\alpha)$ vanish, since $F_{i,0}(\hat{H}_c^i = 0) = 0$. Using eqs. (3.16) and (3.18) in V_2 in eq. (3.3), one immediately gets

$$V = \tilde{g}^{\bar{\alpha}\alpha}\overline{F}_{\bar{\alpha},1}F_{\alpha,1}. \tag{3.21}$$

Finally, going back to non-canonically normalized fields $\hat{L}^\alpha = (T_L^{-1}\hat{L}_c)^\alpha$ gives rise to the full bosonic effective Lagrangian

$$\mathcal{L}_{\text{full}} = \left[(\tilde{g}_{\alpha\bar{\beta}} + O(\epsilon))\partial L^\alpha \partial \bar{L}^{\bar{\beta}} \right] - \left[\epsilon^2 \tilde{g}^{\bar{\alpha}\alpha}\overline{F}_{\bar{\alpha},1}F_{\alpha,1} + O(\epsilon^3) \right], \tag{3.22}$$

which precisely agrees with $\mathcal{L}_{\text{sim}}$, as given by eq. (3.5), at leading order in ϵ.

It is important to remark that eqs. (3.15) and (3.16) hold also when $\hat{L}_c^\alpha \neq 0$, because at $O(\epsilon)$ L^α is undetermined. This implies that light field fluctuations arise also from the expansion of H^i at $O(\epsilon)$ from the field displacements H_1^i in eq. (3.13). Alternatively, we could have proceeded by just using eq. (3.12), valid at the VEV level only, and then expand in a more conventional way $H^i = \langle H^i \rangle + \hat{H}^i$. The terms linear in $\hat{H}^i$ at $O(\epsilon)$ do not vanish any more and the integration over the heavy fields has to be performed. Up to $O(\epsilon^2)$ in the potential, such integration does not present any problem and can straightforwardly be performed, leading to eq. (3.22), in complete agreement with our alternative approach.

4. Supergravity

The analysis of a general SUGRA model closely follows along the lines of the flat SUSY case considered in the previous section, with one extra important requirement, which is a consequence of the universal nature of the gravitational interactions.

The superpotential is taken as in eq. (3.1) and all the assumptions of the flat case continue to hold here. The SUGRA scalar potential is a generalization of eq. (3.2), which reads

$$V = e^{K/M_p^2} \left(g^{\bar{M}N} \overline{F}_{\bar{M}} F_N - 3 \frac{|W|^2}{M_p^2} \right) = M_p^2 e^{G/M_p^2} \left(g^{\bar{M}N} \overline{G}_{\bar{M}} G_N - 3 M_p^2 \right), \qquad (4.1)$$

where, as usual, $F_M = D_M W = \partial_M W + (\partial_M K) W / M_p^2$ is the Kähler covariant derivative and we have also introduced the Kähler invariant function $G = K + M_p^2 \log(|W|^2/M_p^6)$ and its derivatives $G_M = \partial_M G = M_p^2 F_M/W$. M_p is the reduced Planck mass that for simplicity will be set to one in what follows. As remarked above, gravity makes the decoupling of the heavy fields from the light ones harder. This is best seen if we expand the scalar potential in ϵ and analyze the leading term V_0 which reads

$$V_0 = e^K \left(g^{\bar{M}N} \overline{F}_{0,\bar{M}} F_{0,N} - 3|W_0|^2 \right), \qquad (4.2)$$

where

$$F_{0,i} = \partial_i W_0 + (\partial_i K) W_0, \qquad F_{0,\alpha} = (\partial_\alpha K) W_0. \qquad (4.3)$$

It is immediately clear from eqs. (4.2) and (4.3) that even at $O(\epsilon^0)$ there is in general no decoupling between the fields H^i and L^α, as was the case for flat space. As we mentioned in the introduction and can be seen from eqs. (4.3), $F_{0,i} = 0$ is independent of L^α if either $\langle W_0 \rangle \sim O(\epsilon)$ or K is factorizable at leading order. We now separately discuss the two situations.

4.1 Small $\langle W_0 \rangle$

Aside from decoupling, the most important reason to have a small $\langle W_0 \rangle$ is the requirement of a sufficiently light gravitino mass and to ensure a mass hierarchy between the heavy and light fields. When $\langle W_0 \rangle \sim O(\epsilon)$, the expansion of the G_M's is taken as follows:

$$G_i = G_{i,-1} + G_{i,0}, \qquad G_\alpha = G_{\alpha,0}, \qquad (4.4)$$

where we count the powers of ϵ by taking into account the presence of W in the denominator of the G factors. In this way terms of $O(\epsilon^{-1})$ appear, but no poles in ϵ arise, being e^G of $O(\epsilon^2)$. In the particular "Kähler gauge" in which we defined the theory, we have

$$G_{i,-1} = \frac{\partial_i W_0}{W}, \quad G_{i,0} = \partial_i K + \frac{\partial_i W_1}{W},$$
$$G_{\alpha,0} = \partial_\alpha K + \frac{\partial_\alpha W_1}{W}. \qquad (4.5)$$

As in the previous flat space analysis, let us first show that the location of the vacuum is reliably computed in the simple theory, namely that $\partial_\alpha V_{\text{sim}} = \partial_\alpha V + O(\epsilon^3)$, where V_{sim} is

the simple effective potential obtained by setting $H^i = H_0^i$ in K and W. The full e.o.m. for the fields can be written as

$$\partial_M V = G_M V + e^G (G^P \nabla_M G_P + G_M) = 0 \,. \tag{4.6}$$

In eq. (4.6), $G^P = \overline{G}_{\bar{P}} g^{\bar{P}P}$, $\nabla_M G_P = \partial_M G_P - \Gamma^Q_{MP} G_Q$ is the Kähler covariant derivative and $\Gamma^Q_{MP} = (\partial_M g_{P\bar{Q}}) g^{\bar{Q}Q}$ are the holomorphic components of the affine connection. The expansion of $\partial_M G_P$ gives

$$\begin{aligned}
\partial_j G_i &= (\partial_j G_i)_{-2} + (\partial_j G_i)_{-1} + (\partial_j G_i)_0 \,, & \partial_\beta G_i &= (\partial_\beta G_i)_{-1} + (\partial_\beta G_i)_0 \,, \\
\partial_j G_\alpha &= (\partial_j G_\alpha)_{-1} + (\partial_j G_\alpha)_0 \,, & \partial_\beta G_\alpha &= (\partial_\beta G_\alpha)_0 \,.
\end{aligned} \tag{4.7}$$

For simplicity, we do not write the explicit forms of the derivatives of the G's in terms of K and W, being straightforward to derive these expressions. At $O(\epsilon^0)$, $(\partial_i V)_0 = 0$ is satisfied by $(\partial_j G_i)_{-2} = -(G_i)_{-1}(G_j)_{-1} = 0$, i.e. $\partial_i W_0 = 0$, which fix H_0^i. The equations $(\partial_\beta V)_0 = (\partial_\beta V)_1 = 0$ automatically vanish for $H^i = H_0^i$. The equations $(\partial_j V)_1 = 0$ give the linear order displacement of the heavy fields (the analogue of the flat space formula (3.12)):

$$H_1^i = -(\hat{K}^{-1})^i_{\bar{j}} (\overline{G}^{\bar{j}})_0 \,, \tag{4.8}$$

with $\hat{K}^{\bar{i}}_j = g^{\bar{i}k} (\partial_k G_j)_{-1}$ and $(\overline{G}^{\bar{j}})_0 = (\overline{G}_{\bar{M}})_0 g^{\bar{M}i}$, evaluated at $H^i = H_0^i$.

At the shifted vacuum $H_0^i + \epsilon H_1^i$ we have

$$\begin{aligned}
G_i &= O(1), & G_\alpha &= O(1), & G^i &= O(\epsilon), & G^\alpha &= O(1) \,, \\
\nabla_\beta G_i &= O(1), & \nabla_\beta G_\alpha &= O(1), & V &= O(\epsilon^2) \,.
\end{aligned} \tag{4.9}$$

Using eq. (3.18), we also have

$$\begin{aligned}
G^\alpha &= \overline{G}_{\bar{\alpha}} \tilde{g}^{\bar{\alpha}\alpha} + O(\epsilon) \,, \\
\nabla_\beta G_\alpha &= \partial_\beta G_\alpha - \tilde{g}^{\bar{\gamma}\gamma} \partial_\beta g_{\alpha\bar{\gamma}} G_\gamma = (\tilde{\nabla}_\beta G_\alpha) + O(\epsilon) \,,
\end{aligned} \tag{4.10}$$

where $\tilde{\nabla}$ is the covariant derivative constructed in the subspace parametrized by the scalar fields L^α only, namely the one entering in V_{sim}. Finally, using eqs. (4.9) and (4.10), it follows that $\partial_\beta V = \partial_\beta V_{\text{sim}} + O(\epsilon^3)$, where

$$V_{\text{sim}} = e^G \left[\tilde{g}^{\bar{\alpha}\alpha} \overline{G}_{\bar{\alpha}} G_\alpha - 3 \right] \,, \tag{4.11}$$

with heavy moduli frozen at H_0^i.

The equivalence of the simple low-energy effective theory with the full one proceeds along the same lines of the flat space SUSY case discussed before. The expansion of the scalar potential (4.1) is as follows: $V = V_0 + \epsilon V_1 + \epsilon^2 V_2$, with

$$\begin{aligned}
V_0 &= e^G g^{\bar{i}j} \overline{G}_{\bar{i},-1} G_{j,-1} \,, \\
V_1 &= e^G \left[g^{\bar{i}j} \overline{G}_{\bar{j},-1} G_{j,0} + g^{\bar{j}\alpha} \overline{G}_{\bar{j},-1} G_{\alpha,0} + c.c \right] \,, \\
V_2 &= e^G \left[g^{\bar{i}j} \overline{G}_{\bar{j},0} G_{j,0} + g^{\bar{\alpha}\alpha} \overline{G}_{\bar{\alpha},0} G_{\alpha,0} + \left(g^{\bar{i}\alpha} \overline{G}_{\bar{i},0} G_{\alpha,0} + c.c \right) - 3 \right] \,.
\end{aligned} \tag{4.12}$$

The canonically normalized field fluctuations are $\hat{\phi}_c^M$, with $\hat{\phi} = T\hat{\phi}_c$, $\hat{\bar{\phi}} = T^*\hat{\bar{\phi}}_c$ and $g_0 = (T^{-1})^\dagger T^{-1}$. The $\hat{H}_c^i$ are still the proper heavy field fluctuations, as is found by expanding the leading potential term V_0 at quadratic order. The leading mass term, in the parametrization (3.1), is given by $\exp(K)T_H^\dagger M^2 T_H$, with M^2 as in eq. (3.19). In total analogy with the flat case, the integration of $\hat{H}_c^i$ is trivial, just fixing $\hat{H}_c^i = 0$. One has $G_{i,-1} = 0$ and again $G_0^i = 0$ so that

$$V = e^G\Big[\tilde{g}^{\bar{\alpha}\alpha}\overline{G}_{0,\bar{\alpha}}G_{0,\alpha} - 3\Big].\tag{4.13}$$

Going back to non-canonically normalized fields $\hat{L}^\alpha = (T_L^{-1}\hat{L}_c)^\alpha$ gives rise to the full bosonic effective Lagrangian

$$\mathcal{L}_{\text{full}} = \Big[\big(\tilde{g}_{\alpha\bar{\beta}} + O(\epsilon)\big)\partial L^\alpha \partial \bar{L}^{\bar{\beta}}\Big] - \Big[e^G\Big(\tilde{g}^{\bar{\alpha}\alpha}\overline{G}_{0,\bar{\alpha}}G_{0,\alpha} - 3\Big) + O(\epsilon^3)\Big],\tag{4.14}$$

which agrees with $\mathcal{L}_{\text{sim}} = \tilde{g}_{\alpha\bar{\beta}}\partial L^\alpha \partial \bar{L}^{\bar{\beta}} - V_{\text{sim}}$, at leading order in ϵ.

4.2 Almost factorizable Kähler potential

A decoupling between the heavy and light fields is possible also for generic $\langle W_0\rangle$, provided that K is almost factorizable, namely

$$K(\phi, \bar{\phi}) = K_H(H^i, \bar{H}^{\bar{i}}) + K_L(L^\alpha, \bar{L}^{\bar{\alpha}}) + \epsilon K_{\text{mix}}(\phi^M, \bar{\phi}^{\bar{M}}).\tag{4.15}$$

With K as in eq. (4.15), the SUSY equations $F_{0,i} = 0$ do not depend on L^α and lead to solutions of the e.o.m. $(\partial_i V)_0 = 0$ on a generic non-SUSY vacuum with $F_{0,\alpha} \neq 0$. This is expected, since a superpotential of the form (3.1) at $O(\epsilon^0)$ does not depend on L and hence the sufficient conditions for decoupling discussed in [7], namely $K = K_H(H^i, \bar{H}^i) + K_L(L^\alpha, \bar{L}^\alpha)$ and $W = W_H(H^i)W_L(L^\alpha)$ are fulfilled at $O(\epsilon^0)$, with $W_H(H) = W_0(H)$ and $W_L(L) = 1$. As we will see, showing how the decoupling works for a factorizable K is much simpler than the small W_0 case, since it is enough to work out the scalar potential at $O(\epsilon^0)$. The latter reads

$$V_0 = e^{G_0}(G_0^i G_{i,0} + G_0^\alpha G_{\alpha,0} - 3),\tag{4.16}$$

with

$$G_0 = K_H + K_L + \log|W_0|^2\tag{4.17}$$

and $G_{M,0} = \partial_M G_0$. The SUSY conditions $G_{i,0} = 0$ do not depend on L^α and since $\partial_\alpha G_{i,0} = \partial_i G_{\alpha,0} = 0$, they automatically are solutions of the e.o.m. $\partial_i V_0 = 0$, independently of L^α and of $G_{\alpha,0}$. The $O(\epsilon^0)$ e.o.m. for L^α are not necessarily trivial as in the small W_0 situation, since $\partial_\alpha V_0 = 0$ do not automatically vanish. In this case, the vacuum is entirely determined by V_0, which effectively encodes the leading coupling terms between H^i and L^α. The expansion of V_0 in heavy field fluctuations is of the schematic form (3.20). It is straightforward to see from eq. (4.16) that all the mixed derivatives $\partial_i \partial_{\alpha_1} \dots \partial_{\alpha_n} V_0$ vanish [8]. As a result, $\partial_H V$ is at most of $O(\epsilon)$, $\partial_H^2 V$ is of $O(1)$, implying that $\hat{H}_c$ is

$O(\epsilon)$. The leading bosonic effective Lagrangian at $O(\epsilon^0)$ is hence reliably determined by just freezing the heavy fields to their VEV's H_0^i.

It can happen that $\partial_\alpha V_0$ is trivial, if the SUGRA model is of the no-scale type, with

$$G_0^\alpha G_{\alpha,0} = g_L^{\bar\alpha\alpha}\partial_{\bar\alpha}K_L\partial_\alpha K_L = 3\,, \tag{4.18}$$

with g^L the inverse of the Kähler metric $\partial_\alpha\partial_{\bar\alpha}K_L$. When eq. (4.18) is valid, eq. (4.16) simplifies to

$$V_0 = e^{G_0}G_0^i G_{i,0}\,, \tag{4.19}$$

and $\partial_\alpha V_0$ automatically vanish for $G_{i,0} = 0$. The situation is now very similar to the non-SUSY case discussed in section 2. The fields L^α are generally fixed by the $O(\epsilon)$ e.o.m. $\partial_\alpha V_1 = 0$, where V_1 is the next scalar potential term, whose explicit form will not be needed. The expansion in fluctuations of V_0 is as before, with $\hat{H}_c \sim O(\epsilon)$. The leading bosonic effective Lagrangian arises at $O(\epsilon)$ and is determined by just freezing the heavy fields to their VEV's H_0^i. As we will see in appendix C, the large volume models of [17, 18] belong to this class of models.

5. Conclusions

Our results show once again the power of supersymmetry combined with an effective field theory approach, allowing an enormous simplification in the description of complicated field theories with many fields, as the ones appearing in the string landscape. Probably the most important message of our paper is that the often assumed restriction of factorisable Kähler potentials, required to disentangle heavy and light degrees of freedom for superpotentials of the form (1.1), can be relaxed and replaced by the single condition of a small W at the vacuum (when gravity is included). Unless the Kähler potential is abnormally large, as in the large volume models of [17, 18], the smallness of W is phenomenologically required to have a light gravitino, i.e. low-energy SUSY breaking. Our main working assumption is eq. (1.1), which essentially imply that the light fields are stabilized by a dynamics parametrically suppressed compared to the dynamics responsible for the heavy fields stabilization. In the context of effective SUGRA theories with no gauge fields, this assumption is quite natural, ensuring a mass gap between the spectrum of heavy and light fields. This is in fact a common setup in string compactifications with fluxes, where some moduli are stabilized by flux generated superpotentials with relatively strong couplings, the remaining moduli feeling only much weaker non-perturbative generated couplings. We have mentioned in the introduction how our results provide a solid framework for the way complex structure and dilaton fields are treated in KKLT-like models and how the fine-tuning of W_0 is actually more important than previously thought.[11] It is obvious that all these reasonings apply to any other string (or non-string) derived SUGRA model with a superpotential of the form (1.1). For simplicity, we have considered in this paper field theories with a single

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[11]The smallness of W_0 is not necessarily obtained by fine-tuning in theories with an approximate R-symmetry or R-parity, where W_0 can naturally be small. It would be very interesting to find string models with this property.

hierarchy in the field mass spectrum, but generalizations to multiple hierarchies should be straightforward.

The most compelling generalization of our results is the introduction of gauge fields and charged matter and see under what conditions one can still reliably freeze moduli in this context. The issue is particularly important in so called string local models, where all moduli are assumed to be frozen and gravity decoupled, and one constructs (semi)-realistic string models using essentially the much simpler and general model building techniques valid for non-compact spaces (see e.g. [20] for an interesting class of F-theory models of this kind). We hope to come back to this important point in a forthcoming paper [6].

Acknowledgments

A. A flat space model

We show here how the general analysis performed in section 3 works in a concrete example with global $\mathcal{N} = 1$ SUSY. For simplicity, we consider just two fields. The Kähler and superpotential terms are

$$K = \bar{H}H + \bar{L}L - \frac{(\bar{L}L)^2}{4\Lambda^2} + \eta(\bar{H}L + \bar{L}H) \,,$$
$$W = -\frac{M}{2}H^2 + \frac{H^3}{3} + \epsilon\left(\mu^2 L + \frac{Y}{2}HL^2\right) \,, \qquad (A.1)$$

with all parameters taken to be real. Positive definiteness of the metric requires $\eta < 1$. When H is frozen, the model is a simple deformation of the Polonyi model which admits non-SUSY vacua. Due to the R-symmetry breaking HL^2 operator, the non-SUSY vacuum is displaced from the origin in L. The heavy field H enters in W_1 and has a mixing term of $O(1)$ with L due to the last term in K. We take $H_0 = \bar{H}_0 = M$ as solution to the SUSY equation $F_{H,0} = 0$. Neglecting irrelevant terms, the simple effective Kähler and superpotential terms for L are given by

$$K_{\mathrm{sim}} = \bar{L}L - \frac{(\bar{L}L)^2}{4\Lambda^2} \,, \qquad W_{\mathrm{sim}} = \epsilon\left(\mu^2 L + \frac{Y}{2}ML^2\right) \,, \qquad (A.2)$$

resulting in the simple bosonic low-energy Lagrangian

$$\mathcal{L}_{\mathrm{sim}} = \left(1 - \frac{|L|^2}{\Lambda^2}\right)|\partial L|^2 - \epsilon^2 \frac{1}{1 - |L|^2/\Lambda^2}|\mu^2 + YML|^2 \,. \qquad (A.3)$$

The non-SUSY vacuum of V_{sim} is easily found to be

$$L_0 = -\frac{YM\Lambda^2}{\mu^2} \,. \qquad (A.4)$$

It is a simple exercise to show that the vacuum (H_0, L_0) is a (meta-stable) minimum with

$$m_L^2 = m_{\bar{L}}^2 = \epsilon^2 \frac{\mu^{12}}{\Lambda^2 c^2} \,,$$
$$F_L = \epsilon \frac{c}{\mu^2} \,, \qquad\qquad F^L = \epsilon \mu^2 \,, \qquad\qquad\qquad (A.5)$$

where

$$c \equiv \mu^4 - Y^2 M^2 \Lambda^2 \,. \qquad\qquad\qquad (A.6)$$

We now show that the same effective Lagrangian (A.3) is obtained by properly integrating out the canonical heavy degrees of freedom. We start by recomputing the vacuum of the full potential in a series expansion in ϵ. At leading order $H_0 = \bar{H}_0 = M$. At next order, one easily finds

$$H_1(x) = -\frac{Y L(x)^2}{2M} + \frac{\eta(\mu^2 + L(x)MY)}{M\left(1 - \frac{|L|^2(x)}{\Lambda^2}\right)} \,. \qquad\qquad\qquad (A.7)$$

We have written the space-time dependence of the fields in eq. (A.7) to emphasize that this formula is not only valid at the (constant) vacuum but for any light field fluctuation, being L undetermined at this order. At $O(\epsilon^2)$ one gets the same equations of motion for L found with V_{sim}, as well as the shift H_2, the explicit form of which is not necessary. The leading order vacuum (H_0, L_0) coincides then with that obtained before. We can expand the full potential V in small fluctuations $\hat{\phi} = (\hat{H}, \hat{L})^t$ by writing $H(x) = H_0 + \epsilon H_1(x) + \hat{H}(x)$, $L(x) = L_0 + \hat{L}(x)$. The 2×2 metric $g_0 = g(H_0, L_0)$ can be written as $g_0 = (T^{-1})^\dagger T^{-1}$, with T the lower triangular matrix

$$T = \begin{pmatrix} \sqrt{\frac{c}{c-\eta^2\mu^4}} & 0 \\ \frac{\eta\mu^4}{\sqrt{c(c-\eta^2\mu^4)}} & \frac{\mu^2}{\sqrt{c}} \end{pmatrix} \,, \qquad\qquad\qquad (A.8)$$

valid for $\mu^2 > YM\Lambda$. The canonical Lagrangian is obtained by writing $\hat{\phi} = T\hat{\phi}_c$. After some algebra, it is not difficult to see that the canonically normalized fields $\hat{H}_c$ are also mass eigenvectors with a mass given, at leading order, by

$$m_H^2 = m_{\bar{H}}^2 = M^2 \frac{c^2}{(c - \eta^2\mu^4)^2} + O(\epsilon) \,. \qquad\qquad\qquad (A.9)$$

The F-terms for L are as in eq. (A.5) whereas for the heavy field we have

$$F_H = \epsilon\eta\mu^2 \,, \qquad F^H = O(\epsilon^2) \,, \qquad\qquad\qquad (A.10)$$

as expected from our general analysis. Setting $\hat{H}_c = \bar{\hat{H}}_c = 0$ gives the full bosonic effective Lagrangian $\mathcal{L}_{\text{full}}$ for the light fluctuations. It is now a simple, yet algebraically lengthy, exercise to show that at leading order $\mathcal{L}_{\text{full}}$ coincides with $\mathcal{L}_{\text{sim}}$.

B. Fine tuned W: a KKLT-like model with broken SUSY

In this appendix we apply the results of the paper to a KKLT-like SUGRA toy model in a non-SUSY vacuum with $n_H = n_L = 2$. Although the model is pretty simple and contains just four complex fields, instead of hundreds as in realistic string models, it is already sufficiently complicated to make an analytical study a formidable task. For this reason we opt here for a numerical analysis. For obvious reasons, we will not show the equivalence of the entire effective potentials, but we will restrict our attention on the comparison of the vacuum and the scalar mass spectrum. The Kähler and superpotential terms are taken as follows:

$$K = -\log\left[(T + \bar{T})^{3/2} + \xi(S + \bar{S})^{3/2}\right]^2 (Z + \bar{Z})(S + \bar{S}) + \frac{\bar{U}U}{(Z + \bar{Z})^2} - \eta\frac{(\bar{U}U)^2}{(Z + \bar{Z})^4}, \quad \text{(B.1)}$$

$$W = aZ^2 + bZ + S(cZ^2 + dZ + e) + YUZ^2 + \beta Z^2 e^{-\alpha T}. \quad \text{(B.2)}$$

The heavy fields are S and Z, the light ones T and U. The fields S and Z mimic respectively the dilaton and a complex structure modulus of some IIB Calabi-Yau compactification, T represents the overall universal Kähler modulus and U a generic Polonyi field responsible for SUSY breaking. The kinetic mixing between heavy and light fields is provided by the universal α' correction to the volume [21], parametrized by ξ in eq. (B.1) and by the complex structure dependence of the kinetic term for U. The first five terms in the superpotential mimics superpotential terms arising from fluxes and should be identified with W_0 in our general analysis (see e.g. eq.(3.53) of [22]). The term YUZ^2 is a cubic coupling responsible for the main SUSY breaking source in the U direction and $Z^2 \exp(-\alpha T)$ is a non-perturbatively generated coupling, responsible for the stabilization of T, which might arise from euclidean D_3-instantons or gaugino condensation from D_7-branes. These are the small superpotential terms defining W_1. The prefactor of the exponential is a function of the complex structure moduli, which for simplicity we have taken to be just quadratic. It should be stressed that the detailed structure of K and W in eqs. (B.1) and (B.2) is pretty arbitrary and not well motivated; it should just be seen as a simple, yet not trivial, string-inspired example where to concretely apply our results. The parameters entering in eqs. (B.1) and (B.2) have been partially fixed by requiring to have a small positive cosmological constant, $O(1)\,\text{TeV}$ gravitino mass (which fixes the size of W_0 to be 10^{-11}), $S_0 \simeq Z_0 \simeq 10$, small U_0 and $T_0 \sim O(10)$. None of these requirements clearly affect the results of our analysis, but they make our example more "realistic". We have taken[12]

$$a = -2.55 + 10^{-13}, \quad b = 25.5 + 10^{-12}, \quad c = 0.25, \quad d = -2.45, \quad e = -0.5$$
$$\xi = 1, \qquad\qquad \eta = 100, \qquad\qquad \alpha = 1, \quad \beta = -10^{-4}, \quad Y = 0.83 \cdot 10^{-14}. \quad \text{(B.3)}$$

The smallness of Y, required to get a dS vacuum with a sufficiently small cosmological constant, justifies the location of the UZ^2 coupling in W_1. The SUSY VEV's of S and Z,

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[12]In a realistic string setting, the fluxes are quantized and makes the appearance of terms like a and b in eq. (B.3) rather difficult, if not impossible. We do not care about this possible issue. As explained above, we do not pretend our model to mimic a realistic string set-up in any single detail.

	$\langle X \rangle$	$\Delta\langle X \rangle/\langle X \rangle$	F^X	$\Delta F^X/F^X$
S	$10 + 2 \cdot 10^{-13}$	$2 \cdot 10^{-14}$	$8 \cdot 10^{-24}$	—
Z	$10 + 3 \cdot 10^{-13}$	$3 \cdot 10^{-14}$	$-3 \cdot 10^{-23}$	—
T	23.8	$4 \cdot 10^{-15}$	$-3 \cdot 10^{-11}$	$8 \cdot 10^{-14}$
U	0.05	$-3 \cdot 10^{-13}$	$3 \cdot 10^{-10}$	10^{-13}

Table 1: VEV's and F-terms for the fields and their relative shifts, as derived by a numerical analysis. Here and in the main text $\Delta X/X \equiv (X_{\text{full}} - X_{\text{sim}})/X_{\text{full}}$. All quantities are in reduced Planck units.

as given by $\partial_S W_0 = \partial_Z W_0$ are precisely $S_0 = Z_0 = 10$, with $W_0(S_0, Z_0) = 10^{-11}$. In table 1 we report the exact VEV's and F-terms of the fields as numerically computed in the full model and their relative shifts compared to those computed, again numerically, in the simple effective model, with S and Z frozen at their values S_0 and Z_0. The model above belongs to the general class of models studied in the main text with an ϵ roughly $O(10^{-12})$. Keeping two significant digits, the physical masses are

$$m_{H1}^2 = 2.9 \cdot 10^{-2}, \quad m_{H2}^2 = 2.7 \cdot 10^{-2}, \tag{B.4}$$
$$m_{T_i}^2 = 6.6 \cdot 10^{-27}, \quad m_{T_r}^2 = 6.2 \cdot 10^{-27}, \quad m_{U_r}^2 = 1.6 \cdot 10^{-27}, \quad m_{U_i}^2 = 1.6 \cdot 10^{-27},$$

where $H_1 \simeq Z - S$, $H_2 \simeq Z + S$ and the subscript r and i denote real and imaginary field components, respectively. The masses m_{H1}^2 and m_{H2}^2 refer to both components of the complex scalar fields, being SUSY breaking effects negligible. The mass shifts are

$$\frac{\Delta m_{T_i}^2}{m_{T_i}^2} = 3 \cdot 10^{-14}, \quad \frac{\Delta m_{T_r}^2}{m_{T_r}^2} = 2 \cdot 10^{-14}, \quad \frac{\Delta m_{U_i}^2}{m_{U_i}^2} = 9 \cdot 10^{-14}, \quad \frac{\Delta m_{U_r}^2}{m_{U_r}^2} = 10^{-13}. \tag{B.5}$$

Finally, we also report the gravitino mass, the cosmological constant and their relative shifts:

$$m_{3/2}^2 = 1.3 \cdot 10^{-30}, \quad \frac{\Delta m_{3/2}^2}{m_{3/2}^2} = -4 \cdot 10^{-14},$$
$$V_0 = 2.7 \cdot 10^{-32}, \quad \frac{\Delta V_0}{V_0} = 2 \cdot 10^{-11}. \tag{B.6}$$

Notice that being the cosmological constant fine-tuned to be "small", namely of order $10^{-2} m_{3/2}^2$, its relative shift is larger. The latter is inversely proportional to the smallness of V_0.

The values of the F-terms agree pretty well with the scalings in ϵ as expected from our general analysis. The relative shifts are typically smaller than ϵ, due to the fact that the Kähler mixing between the H and L fields coming from (B.1) in the above vacuum are relatively small. This example shows the excellent agreement between the full and the simple theory.

C. Factorizable Kähler potential: a large volume model

Type IIB SUGRA compactifications on CY admit vacua where the internal volume is very large, resulting in so called large volume models [17, 18]. A decoupling occurs in

these models, since they approximately satisfy the conditions for decoupling given in [7].[13] Unfortunately, we did not find an easy way to place such models in a general setting, so that we will focus on an explicit known example ("swiss-cheese" compactification).

Consider a type IIB CY orientifold compactification on $\mathbf{CP}^4_{[1,1,1,6,9]}$. This manifold has $h^{1,1} = 2$ and $h^{2,1} = 272$ Kähler and complex structure moduli, respectively. In order to be able to treat this system, we keep only one complex structure modulus, denoted by Z in the following. As will be clear, our conclusions do not really depend on such drastic simplification. In the usual adiabatic approximation of neglecting flux effects, but keeping the universal α' correction [21], the Kähler potential reads

$$K = -2\log\left[\text{Vol}(T,t) + \xi(S + \bar{S})^{3/2}\right] - \log(S + \bar{S}) - \log(Z + \bar{Z}), \qquad \text{(C.1)}$$

where Vol is the CY volume, which depends on the two Kähler moduli T and t [24]:

$$\text{Vol}(T,t) = \frac{1}{9\sqrt{2}}\left(T_r^{3/2} - t_r^{3/2}\right). \qquad \text{(C.2)}$$

In eq. (C.2) we generally denoted by $X_r = (X + \bar{X})/2$ the real part of a complex field X. The small expansion parameter in this class of models is $\epsilon \equiv 1/\text{Vol} \sim T_r^{-3/2}$, with $T_r \gg 1$ and $t_r \gtrsim 1$. The superpotential is the sum of a flux superpotential $W_0 = W_0(S,Z)$ and a non-perturbative term $W_1 = W_1(S,Z,t)$.[14] The stabilization to large volume requires $W_1 \sim \epsilon$ and hence the superpotential has the form of eq. (3.1):

$$W = W_0(S,Z) + \epsilon W_1(S,Z,t), \qquad \text{(C.3)}$$

where ϵ is introduced in K by redefining $T \to \epsilon^{-2/3}T$. Eq. (C.1) factorizes at $O(1)$, namely

$$K = K_H(S,Z) + K_L(T,t) + \epsilon K_{\text{mix}}(S,T,t), \qquad \text{(C.4)}$$

with $K_H = -\log(4S_r Z_r)$ and $K_L = -2\log\text{Vol}$. Contrary to the KKLT-like models, no fine-tuning on W_0 is required for decoupling. It is straightforward to expand G_M in powers of ϵ. We get

$$G_S = G_{S,0} + O(\epsilon), \qquad G_Z = G_{Z,0} + O(\epsilon),$$
$$G_T = \epsilon^{2/3} G_{T,2/3} + O(\epsilon^{5/3}), \qquad G_t = \epsilon\, G_{t,1} + O(\epsilon^2),$$

with

$$G_{S,0} = -\frac{1}{2S_r} + \frac{\partial_S W_0}{W_0}, \quad G_{Z,0} = -\frac{1}{2Z_r} + \frac{\partial_Z W_0}{W_0},$$
$$G_{T,2/3} = -\frac{3}{2T_r}, \qquad\qquad G_{t,1} = \frac{3\sqrt{t_r}}{2T_r^{3/2}} + \frac{\partial_t W_1}{W_0}.$$

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[13]It has been already noted in ref. [23] that large volume models satisfy an approximate condition of decoupling, but this observation has not been fully exploited there.

[14]Since T is very large, possible non-perturbative terms of the form $\exp(-aT)$ are totally negligible.

The expansion of the scalar potential V is as follows: $V = \epsilon^2 V_2 + \epsilon^3 V_3 + O(\epsilon^4)$, where

$$V_2 = e_2^G \left[g_0^{\bar{Z}Z} \overline{G}_{\bar{Z},0} G_{Z,0} + g_0^{\bar{S}S} \overline{G}_{\bar{S},0} G_{S,0} \right], \tag{C.5}$$

$$e^G = \epsilon^2 e_2^G + O(\epsilon^3), \quad e_2^G = \frac{81|W_0|^2}{2 S_r Z_r T_r^3}. \tag{C.6}$$

It is important to notice that, aside from the overall T-dependence appearing in e_2^G, V_2 depends on S and Z only. A possible non-trivial Kähler moduli dependence of $O(\epsilon^2)$ in V_2 exactly cancels the -3 term in V_2, due to the approximate no-scale structure of the Kähler potential, for which [25]

$$\sum_{i,j=T,t} g^{\bar{j}i} \partial_i K \partial_{\bar{j}} K = 3 + O(\xi\epsilon). \tag{C.7}$$

An effective decoupling between S, Z and the Kähler moduli appears at this order, so that the leading e.o.m. for S and Z admit the SUSY solutions

$$G_{S,0}(S_0, Z_0) = G_{Z,0}(S_0, Z_0) = 0. \tag{C.8}$$

Around the solutions (C.8), all the terms linear in the heavy field fluctuations $\hat{H} = \hat{S}, \hat{T}$ of the form $c_n \hat{H} \hat{L}^n$, where $\hat{L} = \hat{T}, \hat{t}$, $n \geq 1$, vanish. These terms can only arise from V_3, so that schematically we have $O(\epsilon^2)\hat{H}^2 + O(\epsilon^3)c_n \hat{H} \hat{L}^n$, implying that $\hat{H} \sim O(\epsilon)$. Hence integrating out $\hat{H}$ can only result in effective couplings of $O(\epsilon^4)$ or higher and hence we can effectively fix $\hat{H} = 0$. Rather than proceeding as in the main text, by canonically normalizing the fields and so on, in this particular model it turns out to be much simpler to directly compute the full leading effective potential V_3 for T and t. We find

$$V_3 = \frac{27 \left[81\xi_0 |w_0|^2 + 4\sqrt{2t_r} T_r^3 |\partial_t w_1|^2 - 3t_r T_r^{3/2} (\bar{w}_0 \partial_t w_1 + c.c.) \right]}{T_r^{9/2} Z_{r,0} S_{r,0}}, \tag{C.9}$$

where $\xi_0 \equiv \xi S_{r,0}^{3/2}$, $w_0 = W_0(S_0, Z_0)$ and $w_1(t) = W_1(S_0, Z_0, t)$. It is easy to check that V_3 precisely coincides with the simple scalar potential constructed from $W_{\text{sim}} = W(S_0, Z_0)$ and $K_{\text{sim}} = K(S_0, Z_0)$, where the dilaton and the complex structure modulus are frozen at their SUSY values S_0, Z_0 [17]. The scaling in ϵ of the mass spectrum is easily computed. The complex structure modulus, the dilaton and the gravitino mass arise from V_2 and hence

$$m_{3/2} \sim m_S \sim m_Z \sim \epsilon. \tag{C.10}$$

The Kähler structure moduli mass matrix arises from V_3. Given the structure of the kinetic metric, it is simple to see that

$$m_T \sim \epsilon^{3/2}, \quad m_t \sim \epsilon. \tag{C.11}$$

The scalings (C.10) and (C.11) are in agreement with the one reported in the table 1 of [18]. Despite the absence of a hierarchy in the mass scales of the H and L fields, the model is effectively decoupled and the simple naive effective theory, where one freezes S and Z, is reliable.

References

[1] T. Appelquist and J. Carazzone, *Infrared singularities and massive fields*, *Phys. Rev.* **D 11** (1975) 2856.

[2] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219;
M.R. Douglas, *The statistics of string/M theory vacua*, *JHEP* **05** (2003) 046 [hep-th/0303194].

[3] I. Affleck, M. Dine and N. Seiberg, *Dynamical supersymmetry breaking in four-dimensions and its phenomenological implications*, *Nucl. Phys.* **B 256** (1985) 557;

[4] S.P. de Alwis, *On integrating out heavy fields in SUSY theories*, *Phys. Lett.* **B 628** (2005) 183 [hep-th/0506267].

[5] S.P. de Alwis, *Effective potentials for light moduli*, *Phys. Lett.* **B 626** (2005) 223 [hep-th/0506266].

[6] D. Gallego and M. Serone, *An effective description of the landscape — II*, in preparation.

[7] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class. and Quant. Grav.* **21** (2004) 3137 [hep-th/0402046].

[8] A. Achucarro and K. Sousa, *F-term uplifting and moduli stabilization consistent with Kähler invariance*, arXiv:0712.3460;
A. Achucarro, S. Hardeman and K. Sousa, *Consistent decoupling of heavy scalars and moduli in $N = 1$ supergravity*, *Phys. Rev.* **D 78** (2008) 101901 [arXiv:0806.4364].

[9] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **608** (2001) 477] [hep-th/9906070].

[10] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097].

[11] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *De Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240].

[12] H. Abe, T. Higaki and T. Kobayashi, *Remark on integrating out heavy moduli in flux compactification*, *Phys. Rev.* **D 74** (2006) 045012 [hep-th/0606095].

[13] K. Choi, A. Falkowski, H.P. Nilles, M. Olechowski and S. Pokorski, *Stability of flux compactifications and the pattern of supersymmetry breaking*, *JHEP* **11** (2004) 076 [hep-th/0411066].

[14] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [hep-th/0004134].

[15] A. Hebecker and J. March-Russell, *The ubiquitous throat*, *Nucl. Phys.* **B 781** (2007) 99 [hep-th/0607120].

[16] F. Denef and M.R. Douglas, *Distributions of flux vacua*, *JHEP* **05** (2004) 072 [hep-th/0404116].

[17] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of moduli stabilisation in Calabi-Yau flux compactifications*, *JHEP* **03** (2005) 007 [hep-th/0502058].

[18] J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **08** (2005) 007 [hep-th/0505076].

[19] G.H. Golub and C.F. Van Loan, *Matrix computations*, 2$^{\text{nd}}$ edition, Johns Hopkins University, U.S.A. (1989), see section 4.2.3.

[20] C. Beasley, J.J. Heckman and C. Vafa, *GUTs and exceptional branes in F-theory — I*, `arXiv:0802.3391`; *GUTs and exceptional branes in F-theory — II: experimental predictions*, `arXiv:0806.0102`.

[21] K. Becker, M. Becker, M. Haack and J. Louis, *Supersymmetry breaking and α'-corrections to flux induced potentials*, *JHEP* **06** (2002) 060 [`hep-th/0204254`].

[22] D. Lüst, S. Reffert, W. Schulgin and S. Stieberger, *Moduli stabilization in type IIB orientifolds. I: orbifold limits*, *Nucl. Phys.* **B 766** (2007) 68 [`hep-th/0506090`].

[23] A. Achucarro, S. Hardeman and K. Sousa, *F-term uplifting and the supersymmetric integration of heavy moduli*, *JHEP* **11** (2008) 003 [`arXiv:0809.1441`].

[24] F. Denef, M.R. Douglas and B. Florea, *Building a better racetrack*, *JHEP* **06** (2004) 034 [`hep-th/0404257`].

[25] L. Covi et al., *De Sitter vacua in no-scale supergravities and Calabi-Yau string models*, *JHEP* **06** (2008) 057 [`arXiv:0804.1073`].